Loop Quantum Cosmology: A cosmological theory with a view

G A Mena Marugán[1, *]

[1]*Instituto de Estructura de la Materia,*

CSIC, Serrano 121, 28006 Madrid, Spain.

Loop Quantum Gravity is a background independent, nonperturbative approach to the quantization of General Relativity. Its application to models of interest in cosmology and astrophysics, known as Loop Quantum Cosmology, has led to new and exciting views of the gravitational phenomena that took place in the early universe, or that occur in spacetime regions where Einstein's theory predicts singularities. We provide a brief introduction to the bases of Loop Quantum Cosmology and summarize the most important results obtained in homogeneous scenarios. These results include a mechanism to avoid the cosmological Big Bang singularity and replace it with a Big Bounce, as well as the existence of processes which favor inflation. We also discuss the extension of the frame of Loop Quantum Cosmology to inhomogeneous settings.

I. INTRODUCTION

Gravity remains as the only fundamental physical interaction which has not been satisfactorily described quantum mechanically. Leaving aside the belief that all interactions should be unified in a single theory, a strong motivation for a quantum theory of gravity comes from the singularity results of General Relativity [1]. In these classical singularities the predictability breaks down, indicating that the regime of applicability of Einstein's theory has been surpassed and that a new and more fundamental description is needed.

Apart from the ability to cure the classical singularities, one expects that a quantum theory of gravity would open a new window to physics, incorporating phenomena which originate in the quantum realm. Any candidate theory should make this compatible with

*Electronic address: mena@iem.cfmac.csic.es

the infrared behavior of General Relativity, so that no quantum effect alters much spacetime regions like those which we observe [2]. Explaining why our Universe is actually so (semi-)classical constitutes a real challenge to quantum gravity. Previous attempts to construct a quantum formalism for the metric fields (in geometrodynamics) employing a Wheeler-De Witt approach have proven unsuccessful. In the general theory, the approach finds functional analysis and interpretational obstacles which prevent further progress. On the other hand, in simple models where these obstacles can be circumvented, the classical singularities are not fully resolved (e.g., semiclassical states are peaked on trajectories where physical observables eventually diverge [3, 4]). Recently, a different approach to quantize General Relativity has been developed: the theory of Loop Quantum Gravity (LQG) [5, 6]. This theory is based on a nonperturbative canonical formalism of gravity and declares diffeomorphism invariance and background independence to be basic guidelines. Its application to simple cosmological models, generally homogeneous ones, gives rise to the new area of gravitational physics known as Loop Quantum Cosmology (LQC) [2, 7].

II.   LOOP QUANTUM GRAVITY

LQG is a canonical quantization of General Relativity, constructed starting from a Hamiltonian formulation of Einstein's theory. Let us begin by considering a globally hyperbolic spacetime. Introducing then a global foliation, the initial data for the construction of the Lorentzian spacetime metric are contained in the spatial 3-metric and the extrinsic curvature on the considered section of the foliation [8]. If one wants to introduce fermions in this framework, the spatial metric must be replaced with a triad, to which fermions couple directly. This coupling occurs in an internal su(2) index, and respects the invariance of the system under internal rotations, realized then as SU(2) gauge transformations. The spatial metric is recovered as the square of the triad, contracted in the internal indices by means of the Euclidean metric [the Killing-Cartan metric on su(2)]. A canonical set of variables to describe the gravitational phase space is formed, up to numerical factors, by the densitized triad (i.e., the triad multiplied by the square root of the determinant of the spatial metric), and the extrinsic curvature expressed in triadic form (i.e., the extrinsic curvature contracted with the triad in one spatial index).

At this stage, one can replace the extrinsic curvature by a connection valued 1-form, tak-

ing values on the algebra su(2). Classically, this Ashtekar-Barbero connection is a sum of the spin connection compatible with the co-triad and the triadic extrinsic curvature multiplied by an arbitrary positive number γ, known as the Immirzi parameter. By construction, this connection forms a canonical set with the densitized triad, modulo a factor $8\pi G\gamma$ in their Poisson brackets, where G is the Newton constant (from now on, we set the speed of light equal to the unity).

The introduction of the su(2)-connection allows one to incorporate nonperturbative techniques in the description of the system, similar to those developed in gauge field theories, like e.g. for Yang-Mills fields. In particular, the gauge invariant information about the connection is captured in the so-called Wilson loops, constructed from holonomies which describe the parallel transport of spinors around loops. We hence replace the connection by SU(2)-holonomies along (piecewise analytic) edges, where we understand that an edge is an embedding of the closed unit interval in the considered spatial manifold. This replacement involves a 1-dimensional smearing of the connections, and renders the information about them gauge invariant except for the effect of transformations at the end points of the edges. By joining a finite number of edges in those vertices to form a graph [5], and combining the holonomies there so that SU(2) invariance is respected everywhere, one obtains what is usually called a spin network. It is worth noticing that the construction of the holonomies, which contain a line integral of the connections, is made without recurring to any background structure.

Since the most relevant field divergences in our formalism come from the appearance of a 3-dimensional delta function on the Poisson brackets between the connection and the densitized triad, and we have already smeared the connection over one dimension, it seems natural to smear the triad similarly, but now over two dimensions. Given that the densitized triad is a (spatial) vector density, this smearing can be carried out again without employing any background structure. For any (piecewise analytic) surface, we can define the flux of the densitized triad through it, obtaining the desired smearing. Holonomies and fluxes form an algebra under Poisson brackets, which we regard from now on as our basic algebra of functions on the gravitational phase space. From this perspective, the quantization of General Relativity amounts to the representation of this algebra as an algebra of operators acting on a Hilbert space. In addition, we must take into account that the system is subject to a series of constraints, which must be imposed quantum mechanically. These are the

Gauss [or SU(2)] constraint, the diffeomorphisms constraint, and the Hamiltonian or scalar constraint, which express the invariance of the system under SU(2) transformations, spatial diffeomorphisms, and time reparametrizations [5, 6].

An important result for the robustness of the predictions of LQG is a uniqueness theorem about the admissible representations of the holonomy-flux algebra, known as the LOST theorem (after the initials of its authors [9]). Specifically, this theorem states that there exists a unique cyclic representation of that algebra with a diffeomorphism invariant state (interpretable as a vacuum). In total, the choice of the algebra of elementary variables, motivated by background independence, and the status of spatial diffeomorphisms as a fundamental symmetry pick up a unique quantization, up to unitary equivalence and prior to the imposition of constraints.

To gain insight into the kind of quantization adopted in LQG, let us consider the so-called cylindrical functions: complex functions that depend on the connection only via the holonomies along a graph, formed by a finite numbers of edges. Completing this algebra of functions with respect to the norm of the supremum, we obtain a commutative C^*-algebra with identity, where the $*$-relation is provided by the complex conjugation [10, 11]. According to Gel'fand theory, this algebra is then (isomorphic to) the algebra of continuous functions on a certain compact space, called the spectrum. Smooth connections are dense in this space. Besides, the Hilbert space of the representation is just a space of square integrable functions on this Gel'fand spectrum with respect to a certain measure. The LOST theorem guaranties that there exists only one diffeomorphism invariant measure which supports not only a representation of the holonomies, but of the whole holonomy-flux algebra: the Ashtekar-Lewandowski measure, used to construct the representation in LQG. This representation turns out not to be equivalent to a standard one, and therefore leads to physical results which are different from those of other, conventional quantizations. In fact, the representation is not continuous; as a consequence, the connection cannot be defined as an operator valued distribution [5]. Finally, the resulting quantum geometry is discrete, with area and volume operators that have a point spectrum [5].

III. LOOP QUANTUM COSMOLOGY: THE FLAT FRW MODEL

As a paradigmatic example in LQC, let us now apply this type of loop quantization techniques to Friedmann-Robertson-Walker (FRW) cosmologies (namely homogeneous and isotropic spacetimes) with flat spatial sections of $\mathbb{R}^3$ topology and a matter content provided by a homogeneous massless scalar field ϕ, minimally coupled to the metric [3, 4]. We introduce a fiducial triad and an integration cell, adapted to it, to carry out all integrations and avoid in this way divergences due to the homogeneity and noncompactness of the spatial sections. We call V_0 the fiducial volume of this cell. It is possible to check that all physical results are indeed independent of these choices of fiducial elements [3, 4]. Besides, one can fix the gauge freedom so that both the densitized triad and the connection become diagonal. Given the isotropy, they are then totally specified by one single variable each, which we call p and c, respectively. These variables describe the geometry degrees of freedom, vary only in time, and form a canonical pair: $\{c, p\} = 8\pi\gamma G/3$. Classically, they are related with the scale factor a and its time derivative by the formulas $p = V_0^{2/3}a^2$ and $c = \gamma V_0^{1/3}\dot{a}$.

To retain all the gauge invariant information about the su(2)-connection, taking into account the homogeneity, it suffices to consider holonomies along (fiducial) straight edges. Similarly, triads are now smeared across (fiducial) squares. The fluxes are then totally determined by the variable p. Returning to the holonomies, it is easy to check that, for an edge of coordinate length $\lambda V_0^{1/3}$ in any fiducial direction, the matrix elements of the SU(2)-holonomy are linear combinations of exponentials of the form $e^{\pm i\lambda c/2}$. The corresponding configuration algebra is then the linear space of continuous and bounded complex functions in the real line ($c \in \mathbb{R}$), with elements of the form $f(c) = \sum_j f_j e^{i\lambda_j c/2}$. It is well known that the completion of this algebra with the supremum norm is just the Bohr C^*-algebra of almost periodic functions [11]. Its Gel'fand spectrum is the Bohr compactification of the real line, $\mathbb{R}_{\text{Bohr}}$. This compactification can be seen as the set of group homomorphisms from the group of real numbers (with the sum) to the multiplicative group $\mathbb{C}$ of complex numbers of unit norm. Indeed, for every real number c we have a homomorphism $x_c : \mathbb{R} \to \mathbb{C}$ of this kind, namely $x_c(\lambda) = e^{i\lambda c/2}$. Moreover, it is possible to see that the real line is actually dense in $\mathbb{R}_{\text{Bohr}}$, using the fact that our initial configuration algebra separates points $c \in \mathbb{R}$ [10, 11].

The operation $x\tilde{x}(\lambda) = x(\lambda)\tilde{x}(\lambda)$ provides a commutative group structure in $\mathbb{R}_{Bohr}$. Since

the group $\mathbb{R}_{\text{Bohr}}$ is compact, it has a (unique) invariant Haar measure. The functions on $\mathbb{R}_{\text{Bohr}}$ consisting in the evaluation at a real point μ form an orthonormal basis in the corresponding Hilbert space of square integrable functions with the norm defined by that measure [11]. We designate each element in this basis with a ket $|\mu\rangle$. This basis allows us to pass from our configuration representation, in which holonomies act by multiplication, to a "momentum" representation in which the triad has a multiplicative action [10, 11]. Calling $N_\lambda = e^{i\lambda c/2}$, this "momentum" representation is given by $\hat{p}|\mu\rangle = (4\pi\gamma G/3)\mu|\mu\rangle$ and $\hat{N}_\lambda|\mu\rangle = |\mu + \lambda\rangle$ (we set $\hbar = 1$). Clearly, the basis $\{|\mu\rangle;\ \mu \in \mathbb{R}\}$ is uncountable, and therefore the Hilbert space is nonseparable. Nevertheless, normalizable states can get nonvanishing contributions only from a countable subset of states $|\mu\rangle$; otherwise their norm would not be finite. This "momentum" representation is the one usually employed in LQC. It is worth remarking that the representation fails to be continuous, owing to the discrete norm on the Hilbert space, $\langle\mu|\tilde{\mu}\rangle = \delta_\mu^{\tilde{\mu}}$. As a result, a connection operator does not truly exist, and the representation is inequivalent to the Wheeler-De Witt one, in total parallelism with the situation found in LQG for the general case.

Although homogeneity ensures that the diffeomorphisms constraint is satisfied, and the $SU(2)$ constraint has been removed by gauge fixing, the system is still subject to a Hamiltonian constraint, which must be imposed now quantum mechanically. In order to introduce a Hamiltonian constraint operator, there are essentially two building blocks which must be defined in terms of our elementary operators $\hat{p}$ and $\hat{N}_\lambda$. First, we need an operator to represent the phase space function $t(p) = \text{sign}(p)/\sqrt{|p|}$. This function contains all the p-dependence of the (nondensitized) triad of the model. Note that this triad diverges at the Big Bang, where the variable p vanishes. Correspondingly, the operator representing $t(p)$ cannot be defined exclusively from $\hat{p}$ using the spectral theorem: $\hat{p}$ has a point spectrum which contains the zero, and hence its inverse operator is not well defined. But it is possible to construct a regularized triad operator using commutators with holonomies, in addition to $\hat{p}$ [12]: $\widehat{t(p)} = 3(\hat{N}_{-\bar{\mu}}|\hat{p}|^{1/2}\hat{N}_{\bar{\mu}} - \hat{N}_{\bar{\mu}}|\hat{p}|^{1/2}\hat{N}_{-\bar{\mu}})/(4\pi\gamma G\bar{\mu})$. In principle, $\bar{\mu}$ may take any real value. It will be fixed later on in our discussion. The resulting operator is diagonal in the considered basis of μ-states. Furthermore, it turns out to be bounded from above, so that, in particular, the classical divergence disappears. Actually, our regularized operator annihilates the kernel of $\hat{p}$.

The other block that we need is the SU(2)-curvature operator. Recall that the connection

operator is not well defined, therefore we cannot use it to construct the curvature. Nonetheless, it is possible to determine it using a square loop of holonomies. We use again edges of fiducial length $\bar{\mu}V_0^{1/3}$. Classically, the expression of the curvature would be recovered exactly in the limit of zero area, when $\bar{\mu}$ tends to zero. However, this limit is not well defined in LQC. The idea is to shrunk the square up to the minimum physical area Δ allowed in LQG, where the spectrum of the area operator is discrete [4]. This introduces a certain nonlocality in the formalism, and turns the parameter $\bar{\mu}$ into a state dependent quantity, since the physical area depends on the particular state under consideration. Explicitly, the relation that must be satisfied for each state is $\bar{\mu}^2|p| = \Delta$. At this stage, it is convenient to relabel the basis of μ-states as a basis of volume eigenstates, introducing an affine parameter v for the translation generated by $\bar{\mu}c/2$ [4]. By construction, we then get that $\hat{N}_{\bar{\mu}}|v\rangle = |v+1\rangle$. The parameter v is related with the physical volume of the fiducial cell, $V = p^{3/2}$, by the formula $v = \text{sign}(p)V/(2\pi\gamma G\sqrt{\Delta})$.

Finally, for the quantization of the matter field ϕ, we use the standard Schrödinger representation. So, the kinematical Hilbert space is the tensor product of the gravitational space of LQC and the standard one for matter. With a suitable factor ordering and choice of densitization [13], one then gets a Hamiltonian constraint of the form $\hat{H} = -6\hat{\Omega}^2 + \hat{P}_\phi^2$, where $\hat{P}_\phi$ is the momentum operator of the matter field, and acts by differentiation (namely, $\hat{P} = -i\partial_\phi$). The gravitational part of the constraint is given by the operator $\hat{\Omega}^2$. Remarkably, this constraint leaves invariant the zero-volume state $|v = 0\rangle$, as well as its orthogonal complement. Therefore, the analog of the classical singularity is removed in practice from the Hilbert space, and we can restrict all physical considerations to its complement. In this sense, the singularity gets resolved at a kinematical level. Moreover, the operator $\hat{\Omega}^2$ has an action of the following type: $\hat{\Omega}^2|v\rangle = f_+(v)|v+4\rangle + f(v)|v\rangle + f_-(v)|v-4\rangle$. Here, the real functions $f_+(v)$ and $f_-(v)$ have the outstanding property that they vanish in the respective intervals [-4,0] and [0,4] [13]. Thus, the action of $\hat{\Omega}^2$ preserves each of the subspaces of the gravitational Hilbert space obtained by restricting the label of the v-states to any of the semilattices $\mathcal{L}_\varepsilon^\pm = \{v = \pm(\varepsilon + 4n); n \in \mathbb{N}\}$, where $\varepsilon \in (0,4]$. Then, each of these semilattices provides a superselection sector. In each sector, the orientation of the triad is definite (v does not change sign) and $|v|$ has an strictly positive minimum, equal to ε. For concreteness, we choose sectors with $v > 0$ from now on.

On the other hand, it is possible to show that, on each sector, the operator $\hat{\Omega}^2$ has

a nondegenerate absolutely continuous spectrum equal to the positive real line [13, 14]. Recalling its action, this gravitational constraint operator might be understood as a second-order difference operator. But its eigenfunctions are entirely determined by their value at ε, point from which they can be constructed by solving the eigenvalue equation. In this sense, the gravitational constraint operator leads to a No-Boundary description, in which the eigenstates which encode the information about the quantum geometry are all determined without the need to introduce any boundary condition in the region around the origin. We also notice that, up to a global phase, these eigenfunctions $e_\delta^\varepsilon(v)$ are real, since so is the gravitational constraint operator.

With such eigenfunctions, one easily finds the solutions to the Hamiltonian constraint, which have the form $\psi(v, \phi) = \int_0^\infty d\delta \, e_\delta^\varepsilon(v)[\psi_+(\delta)e^{i\sqrt{6\delta}\phi} + \psi_-(\delta)e^{-i\sqrt{6\delta}\phi}]$. The scalar field ϕ plays the role of an emergent time. Then, physical states can be identified (e.g.) with the positive frequency solutions $\psi_+(\delta)$ that are square integrable over the spectral parameter $\delta \in \mathbb{R}^+$ [13]. A complete set of Dirac observables is formed by $\hat{P}_\phi$ and $|\hat{v}|_{\phi_0}$, the latter being defined by the action of the volume operator when the field equals ϕ_0. On the Hilbert space of physical states specified above, these observables are self-adjoint operators.

IV. THE BIG BOUNCE

In the previous section we have completed the quantization of the flat FRW with a homogeneous massless scalar field. In order to analyze the physical predictions of this quantum theory, we will consider now the evolution of (positive frequency) quantum states with a semiclassical behavior. We study *Gaussian-like* states which, at an instant $\phi = \phi_0$ in the region of large emergent times $\phi_0 \gg 1$, are peaked on certain values of the elementary observables of the model, namely, the matter momentum, $P_\phi = P_\phi^0$, and the physical volume, $v = v^0$. We restrict our attention to states with large values of P_ϕ^0 and v^0 [3, 4]. The numerical analysis of the quantum evolution unveils an outstanding phenomenon in these states: the Big Bang singularity is resolved dynamically and is replaced by a bounce that connects the universe with another branch of the evolution, dictated again by the equations of General Relativity. This mechanism to elude the cosmological singularity is known as the Big Bounce [3, 4].

The numerical studies show that the considered semiclassical states remain peaked on a

well defined trajectory during the whole evolution. On these states, the Big Bounce does not occur in a genuinely quantum region where one were to loose an effective notion of geometry and spacetime. The trajectory deviates from the one predicted by General Relativity only when the matter energy density ρ becomes of the order of one percent of a critical density, ρ_{crit}. This scale for the onset of corrections is of the Planck order and universal: it is the same for all the semiclassical states which suffer the bounce. Explicitly, the critical density is $\rho_{\text{crit}} = (\sqrt{3}/32\pi^2\gamma^3 G^2) \approx 0.41\rho_{\text{Planck}}$, where ρ_{Planck} is the Planck density. For densities close to the critical one, i.e., in the regime close to the bounce, gravity behaves as a repulsive force owing to the effects of quantum geometry [2].

In addition, the trajectory followed by the peak of these states matches an effective dynamics, which has been deduced in detail (under certain assumptions on the family of states under consideration) using techniques of geometric quantum mechanics [15]. The agreement between the numerical simulations and the predictions of this effective dynamics is remarkable. In particular, the effective dynamics predicts a bounce precisely when the matter density reaches the critical value ρ_{crit}. Further support to the role played by this critical density comes from the study of the operator which represents the matter density in the quantum theory. It is possible to prove that it has a bounded spectrum, the bound being given again by ρ_{crit}. Then, the overview picture is that the emergence of important quantum geometry effects in this model is controlled by the value of the matter energy density. When this density approaches the Planck scale, quantum geometry phenomena enter the scene, preventing that it keeps on increasing and consequently avoiding the collapse into a cosmological singularity. It is worth pointing out that these quantum phenomena can be relevant even in regions which one would not consider to belong to the deep Planck regime. For instance, the volume v at the bounce is proportional to the value of the matter field momentum, which is conserved in the evolution. Hence, when the bounce occurs, the volume can be as large as desired.

The presence of a quantum bounce is actually generic in this quantum model, with implications that exceed the restriction to the discussed class of semiclassical states. We have already commented that the eigenfunctions of the gravitational constraint operator are real (up to a global phase). Studying the Wheeler-De Witt limit of this operator, one can prove that its eigenfunctions lead in fact to positive frequency solutions with ingoing and outgoing components of equal amplitude in this limit [13] (see [16] for the case of a specific

superselection sector).

Furthermore, the Big Bounce mechanism is not restricted just to the flat FRW model with a massless scalar field, but is rather general. On the one hand, assuming the validity of the effective dynamical equations for other matter contents (assumption that is supported by the numerical analyses carried out so far), one can show that all strong singularities (á la Królak) are resolved in flat FRW for any kind of matter [17]. Only Type II and Type IV singularities may remain [17], but these singularities can be considered physically harmless, since geodesics can be extended beyond them (then, sufficiently strong in-falling detectors can survive these singularities). On the other hand, similar conclusions about the occurrence of the Big Bounce have been reached in other FRW models quantized in LQC. These include the flat model with negative cosmological constant [18], the closed model [19], the open model [20] (some problems of the treatment presented in that reference can be solved with the techniques of [21]), and the flat model with positive cosmological constant (recently studied by Ashtekar and Pawłowski), all of them with a homogenous scalar matter field present as well. For the flat FRW universe with negative cosmological constant and the closed FRW model, the classical evolution leads to a Big Crunch (i.e., the universe recollapses into a cosmological singularity). In the quantum theory, this Big Crunch is also resolved via a Big Bounce, like in the case of the Big Bang. In all cases, there exists an upper bound for the matter energy density, which is given again exactly by $\rho_{\rm crit}$, and the infrared regime shows an outstanding agreement with General Relativity.

In spite of some statements that have appeared in the literature of LQC [17], the effective equations for flat FRW in the presence of generic matter do not necessarily lead to an asymptotically de Sitter behavior if a vanishing or a divergent value of the scale factor a were to be approached in the evolution, without further assumptions. The confusion comes from the consideration of the identity $\ln(\rho/\rho_0) = \int_{a_0}^{a}[1 + w(\tilde{a})]d\tilde{a}/\tilde{a}$, deduced from the conservation equation for the matter energy density, and where a_0 is a reference value for the scale factor, $\rho_0 = \rho(a_0)$, and $w(a)$ is the ratio between the pressure and the energy density of matter. In fact, the convergence of the above integral when $a \to \infty$ does not need that $w(a)$ tend to minus the unity [22] (the value that would correspond to a de Sitter regime). Besides, even if the energy density is required to be positive and bounded from above by the critical density, one may have a vanishing limit for it. Then, the considered integral would diverge in that limit, allowing for values of w different from minus one [22].

This situation might be found in the limit of null a.

Similar conclusions about the actual resolution of cosmological singularities have been reached in other homogeneous scenarios which contain anisotropies. LQC has been implemented successfully to completion in Bianchi models of type I [14, 23–26], type II [21], and type IX [27]. The analysis, complemented with numerical simulations in these models, confirms the Big Bounce scenario, though now there may exist bounces in different scale factors, since the spatial directions are not all equivalent in view of the anisotropy. Together with the BKL conjecture [28], these results suggest a generic resolution of spacelike singularities in LQC. Recall that the BKL conjecture says that spatial derivatives can be neglected against time derivatives when one approaches a spatial singularity, so that the dynamics at any point can be approximated locally by a homogeneous model (i.e., a Bianchi model).

The anisotropic model which has received more attention is Bianchi I. Recently, this model has been studied thoroughly with the prescription put forward by Ashtekar and Wilson-Ewing [24] to determine the lengths of the edges used to define the holonomies that enter the Hamiltonian constraint. Actually, it has been shown that this prescription is characterized by the requirement that the action of the corresponding holonomies produce a constant shift in the (absolute value of the) physical volume [25]. In addition, one can see that the initial value problem for the quantum evolution is well posed even in vacuo: an infinite though countable set of initial data on the section of minimum physical volume suffices to fix the solution to the constraint [26]. This allows one to identify the space of solutions and construct from it the Hilbert space of physical states. Finally, while the superselection sectors for the physical volume are the same as in flat FRW, the sectors for the anisotropies turn out to have a rather different structure. They are still discrete, and different triad orientations are not mixed, but these sectors are dense on the (positive) real line, instead of being formed by points separated by a constant distance [25].

Further support to the Big Bounce scenario in anisotropic models, using the prescription of [24] in the quantization, comes from the extrapolation of the effective dynamical equations, assuming their validity for Bianchi I. These effective equations guarantee that the directional Hubble rates, the expansion, and the shear scalar (of comoving observers) are all bounded from above in the evolution [29], preventing in this way the formation of dangerous singularities. Contrary to what one could naively expect [29], however, the bounded nature of these physical quantities cannot be extrapolated to the genuine quantum theory. The

reason is that a bounded function on phase space (namely, one of our physical quantities in the effective theory) is not always represented by a bounded operator. One can ensure that the corresponding operator is bounded only when it can be defined in terms of a set of commuting elementary ones via the spectral theorem. But in generic situations this is not the case. Similarly, an unbounded function on phase space may be represented by a bounded operator. In addition, when superselection sectors enter the scene, the physically relevant spectra of our elementary operators do not coincide with the range of their classical analogs, therefore introducing limitations to the domain of applicability of the effective equations. For instance, the spectrum of the (absolute value of the) physical volume is bounded from below by a positive number on each of our superselection sectors. This invalidates the analysis of the limit of vanishing volume in the effective equations.

V. INFLATION

The effects of quantum geometry in the *early universe* are important not only to elude cosmological singularities, but also to build up a satisfactory inflationary scenario. In standard cosmology, one generally needs a fine tuning of the initial conditions or the inflationary parameters in order to reach enough inflation to explain the observed universe, with at least 68 e-foldings. Recent results in LQC indicate that, on the contrary, the quantum phenomena that accompany the Big Bounce render natural an inflationary process with this number of e-foldings.

Let us consider a flat FRW model with an inflaton field with positive kinetic energy. The effective equations for this type of models in LQC imply a series of interesting properties [2, 30]. Firstly, the Hubble parameter is bounded from above. Besides, when the inflaton potential is nonnegative and bounded from below, the time derivatives of the inflaton and of the Hubble parameter turn out to be bounded from above in absolute value. In addition, for potentials which are unboundedly large when the inflaton field approaches plus/minus infinity, there exists an upper/lower bound on the value of this inflaton. On the other hand, and more remarkably, there always exists a phase of superinflation [31, 32] after the bounce, in which the Hubble parameter increases from zero up to its maximum value. This phenomenon of superinflation is robust in LQC, and appears even in the absence of potential. Nevertheless, one can show that the superinflation epoch alone does not yield

sufficient e-foldings in generic situations.

For concreteness, we will focus our discussion on the inflationary potential $m^2\phi^2$, with a mass of the order of 10^{-6} in Planck units, which is the phenomenologically preferred value. We can calculate the probability of getting more than 68 e-foldings, starting with equiprobability for every unconstrained set of initial data. Taking the bounce as the most natural instant to define initial data, and imposing the Hamiltonian constraint, we obtain on the space of physical data a measure of the form $\sqrt{1 - F_{\text{Bounce}}}\, d\phi_{\text{Bounce}}\, dv_{\text{Bounce}}$, where ϕ_{Bounce} and v_{Bounce} are the values of the field and the physical volume on the bounce section, respectively. Besides, F_{Bounce} is the fraction of the matter energy density which is due to the potential at that moment. On the other hand, we recall that $|\phi_{\text{Bounce}}|$ is bounded, because the studied potential grows unboundedly. Then, it is possible to see that, if $F_{\text{Bounce}} > 1.4 \cdot 10^{-11}$, the superinflationary phase either provides the desired number of e-foldings by its own or supplements them by funneling the solutions to initial conditions such that there is a sufficiently long period of slow-roll inflation [30]. It is straightforward to compute that the relative probability for a value of F_{Bounce} in this range is greater than 0.99. These conclusions are not sensitive to reasonable changes, e.g., in the inflaton mass or the form of the potential. Therefore, the effective equations that arise in LQC solve the fine tuning problems for inflation.

VI.   INHOMOGENEOUS MODELS

Inhomogeneous models have been considered lately in the framework of LQC in order to extend the applicability of this quantization scheme beyond the simple FRW or Bianchi cosmological spacetimes and with the aim at obtaining predictions that, eventually, might be contrasted with observations. The quantum analysis of this type of models has been carried out adopting a hybrid approach [25, 26, 33, 34], which combines a loop quantization of the subspace of homogeneous solutions of the system (or rather of their geometry) and a Fock quantization of the matter fields and inhomogeneous gravitational waves contained in the model. This hybrid approach is based on the assumption that there exists a hierarchy of quantum phenomena, so that the most relevant effects of the loop quantum geometry are those that affect the sector of homogeneous degrees of freedom.

For a series of cosmological models, the ambiguity in the selection of a Fock quantiza-

tion for the inhomogeneities can be removed by appealing to some recent theorems which guarantee the uniqueness of the choice of both a field description (among a reasonable set of possibilities) and a Fock representation for it [35–38]. The uniqueness is ensured if the quantization respects certain conditions on the unitarity of the dynamics as well as the invariance of the vacuum under the spatial symmetries of the field equations. As a consequence, the predictions of the hybrid quantization are robust, in the sense that they are not affected by the typical ambiguities which plague the quantization of systems with an infinite number of degrees of freedom. Cosmological systems where these uniqueness results have been proven, and the hybrid quantization is at hand, include the Gowdy spacetimes [39] (with different spatial topologies and possibly containing scalar fields), and fields and perturbations around closed FRW models.

In the inhomogeneous cosmologies that have been studied in this way, the loop quantization of the homogeneous sector suffices to resolve the cosmological singularities at the kinematical level, in a similar manner as it happens for the flat FRW model. It is remarkable that, in spite of the field character of these systems, the quantization can been carried out to completion. Moreover, the Hilbert space of physical states that one attains with this hybrid approach is such that one recovers the Fock description of the inhomogeneities of the system, providing support to this conventional quantum treatment and proving its compatibility with the loop quantization.

In particular, the hybrid quantization process has been discussed in detail in the (vacuum) Gowdy model with 3-torus topology [25, 34], probably the simplest inhomogeneous cosmology. In this case, the effective dynamics that one obtains extrapolating the results deduced in FRW scenarios has been studied both numerically and (partially) analytically [40] (although the prescription used to determine the lengths of the edges for the holonomies is not that in [24]). The analysis confirms the presence of the Big Bounce, which happens in all the three spatial directions of this anisotropic model. The bounce occurs typically at values of the fluxes variables (i.e., the p-variables for each of the three spatial directions) which are at least a 13 per cent of those found in the absence of inhomogeneities [40]. This proves that the inhomogeneities do not have a drastic effect that could alter substantially the Big Bounce, e.g., driving the bounce into the deep Planck regime. In addition, the numerical studies have considered the change in amplitude of the inhomogeneous modes (which describe linearly polarized gravitational waves) between the two asymptotic regions

of the effective trajectories, which correspond respectively to a contracting and an expanding universe. One can take a statistical average, disregarding phases in the mode decomposition of the inhomogeneities. Actually, it is possible to see that, in the sector of the space of solutions where the inhomogeneities dominate the bounce dynamics, the change in the amplitudes is antisymmetric with respect to the phase. Then, in average, the amplitudes are statistically preserved through the bounce. On the other hand, in the sector where the vacuum dynamics is approximately valid around the bounce, the change in the amplitudes is positive in average [40]. Although the scenario is not completely physical, since the considered inhomogeneities are not all those allowed in the most general cosmological setting, the found behavior may indicate a LQC mechanism that removes low amplitudes through the bounce.

VII. CONCLUSION

The increasing attention paid recently to the quantization of cosmological systems using loop techniques has crystalized in the foundation of a new branch of gravitational physics. This new formalism of LQC allows one a rigorous control on the mathematical and interpretational aspects of quantum cosmology, providing significance and robustness to the predictions in an area where they are rarely falsifiable in a direct way. Moreover, in doing so, LQC has opened new views to the quantum phenomena of the *early universe*. It leads to a new paradigm for cosmology in which the Big Bang singularity is resolved and replaced with a Big Bounce. Remarkably, this Big Bounce respects the semiclassicality of the universe, connecting two branches whose asymptotic behavior is well described by General Relativity. On the other hand, LQC renders inflation a natural process, suppressing the need for a fine tuning in the initial conditions or parameters of the inflationary era. In addition, LQC might supply a mechanism to remove low amplitudes from the (primordial) inhomogeneities in cosmology. And it suggests new settings for the consideration of initial conditions on the inhomogeneities, which would not be imposed anymore in the region around the classical singularity. Further research is needed to explore the consequences in issues such as cosmological perturbations, primordial fluctuations, or the power spectrum of anisotropies in the cosmic background. These are exciting topics that can provide a suitable arena where the predictions of LQC might be compared with observations.

Acknowledgments

References

[1] Hawking S and Ellis G F R 1973 *The Large Scale Structure of Space-Time* (Cambridge: Cambridge University Press)

[2] Ashtekar A 2010 *Preprint* arXiv:1005491

[3] Ashtekar A, Pawłowski T and Singh P 2006 *Phys. Rev. Lett.* **96** 141301
Ashtekar A, Pawłowski T and Singh P 2006 *Phys. Rev.* D **73** 124038

[4] Ashtekar A, Pawłowski T and Singh P 2006 *Phys. Rev.* D **74** 84003

[5] Ashtekar A and Lewandowski J 2004 *Classical Quantum Gravity* **21** R53

[6] Thiemann T 2007 *Modern Canonical Quantum General Relativity* (Cambridge: Cambridge University Press)

[7] Bojowald M 2008 *Living Rev. Rel.* **11** 4

[8] Wald R M 1984 *General Relativity* (Chicago: University of Chicago Press)

[9] Lewandowski J, Okolow A, Sahlmann H and Thiemann T 2006 *Comm. Math. Phys.* **267** 703

[10] Mena Marugán G A 2009 *AIP Conf. Proceedings* **1130** 89

[11] Velhinho J M 2007 *Classical Quantum Gravity* **24** 3745

[12] Ashtekar A, Bojowald M and Lewandowski J 2003 *Adv. Theor. Math. Phys.* **7** 233

[13] Martín-Benito M, Mena Marugán G A and Olmedo J 2009 *Phys. Rev.* D **80** 104015

[14] Martín-Benito M, Mena Marugán G A and Pawłowski T 2008 *Phys. Rev.* D **78** 064008

[15] Taveras V 2008 *Phys. Rev.* D **78** 064072

[16] Ashtekar A, Corichi A and Singh P 2008 *Phys. Rev.* D **77** 024046

[17] Singh P 2009 *Classical Quantum Gravity* **26** 125005

[18] Bentivegna E and Pawłowski T 2008 *Phys. Rev.* D **77** 124025

[19] Ashtekar A, Pawłowski T, Singh P and Vadersloot K 2007 *Phys. Rev.* D **75** 024035

[20] Vandersloot K 2007 *Phys. Rev.* D **75** 023523

[21] Ashtekar A and Wilson-Ewing E 2009 *Phys. Rev.* D **80** 123532

[22] Jaramillo D 2010 *Private Communication*

[23] Chiou D 2007 *Phys. Rev.* D **75** 024029

[24] Ashtekar A and Wilson-Ewing E 2009 *Phys. Rev.* D **79** 083535

[25] Garay L J, Martín-Benito M and Mena Marugán G A 2010 *Phys. Rev.* D **82** 044048

[26] Martín-Benito M, Mena Marugán G A and Wilson-Ewing E 2010 *Phys. Rev.* D **82** 084012

[27] Wilson-Ewing E 2010 *Phys. Rev.* D **82** 043508

[28] Belinskiĭ V A, Khlatnikov I M and Lifshitz E M 1970 *Adv. Phys.* **19** 525

[29] Corichi A and Singh P 2009 *Phys. Rev.* D **80** 044024

[30] Ashtekar A and Sloan D 2010 *Phys. Lett.* B **694** 108

[31] Bojowald M 2002 *Phys. Rev. Lett.* **89** 261301

[32] Singh P 2006 *Phys. Rev.* D **73** 063508

[33] Martín-Benito M, Garay L J and Mena Marugán G A 2008 *Phys. Rev.* D **78** 083516

[34] Mena Marugán G A and Martín-Benito M 2009 *Int. J. Mod. Phys.* A **24** 2820

[35] Corichi A, Cortez J, Mena Marugán G A and Velhinho J M 2006 *Classical Quantum Gravity* **23** 6301

[36] Cortez J, Mena Marugán G A and Velhinho J M 2007 *Phys. Rev.* D **75** 084027

[37] Cortez J, Mena Marugán G A and Velhinho J M 2010 *Phys. Rev.* D **81** 044037

[38] Cortez J, Mena Marugán G A, Olmedo J and Velhinho J M 2010 *JCAP* **10** 030

[39] Gowdy R H 1974 *Ann. Phys.* **83** 203

[40] Brizuela D, Mena Marugán G A and Pawłowski T 2010 *Classical Quantum Gravity* **27** 52001